Circles- Marking Scheme

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = ...$ This involves combining the equations to eliminate θ or find θ

The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $...r^2\theta = 11$ and ones that simplify to $..r = ..r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

....

The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

2.

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of *PA* is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=...$ to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=(\sqrt{20})^2$

(c)

M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$ or, **in the alternative**, for solving their $(4k-34)^2-4\times 5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2 - 4\times5\times(k^2-10k+54)=0 \Rightarrow k=...$ Allow use of a calculator here to find roots.
Award if you see use of correct formula but it would be implied by ± correct roots
A1: $k=-19$ only

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding k

A1: $k=-19$ only

Other method are possible using trigonometry.

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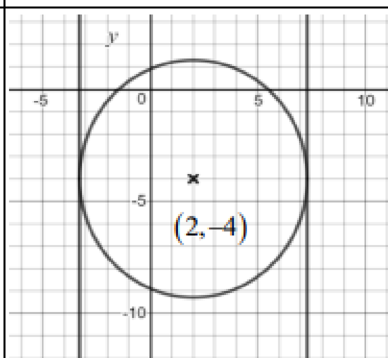
3.

Question	Scheme	Marks	AOs
10(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract 'r' from '2'	M1	3.1a
	$k=2\pm\sqrt{28}$	A1ft	1.1b
		(2)	
			(5 marks)

(a)

M1: Attempts to complete the square. Look for $(x \pm 2)^2 + (y \pm 4)^2 ...$

If a candidate attempts to use $x^2 + y^2 + 2gx + 2fy + c = 0$ then it may be awarded for a centre of $(\pm 2, \pm 4)$ Condone $a = \pm 2, b = \pm 4$

A1: Centre $(2, -4)$ This may be written separately as $x = 2, y = -4$ BUT $a = 2, b = -4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y = -4$ into circle equation and finds x/k by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x = k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2 + 8y + k^2 - 4k - 8 = 0$ and use the fact that this would have one root. E.g. $b^2 - 4ac = 0 \Rightarrow 64 - 4(k^2 - 4k - 8) = 0 \Rightarrow k = ..$ Condone slips but the method must be sound.

A1ft: $k = 2 + \sqrt{28}$ and $k = 2 - \sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k = 2 \pm 5.29 \rightarrow k = 7.29, k = -3.29$

Accept just $2 \pm \sqrt{28}$ or equivalent such as $2 \pm 2\sqrt{7}$

Condone $x = 2 + \sqrt{28}$ and $x = 2 - \sqrt{28}$ but not $y = 2 + \sqrt{28}$ and $y = 2 - \sqrt{28}$

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4.

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = ...$	M1	1.1b
	(i) Centre $(3, -5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Substitute $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$	M1	3.1a
	Collects terms to form correct 3TQ $(1 + k^2)x^2 + (10k - 6)x + 9 = 0$	A1	1.1b
	Attempts $b^2 - 4ac...0$ for their a, b and c leading to values for k "$(10k-6)^2 - 36(1+k^2)...0$" $\rightarrow k = ..., ...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2 - 4ac > 0$ and chooses the outside region (see note) for **their** critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k < 0, k > \frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

(a)

M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$

This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$

(i) A1: Centre $(3,-5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant 0$ or even with greater than symbols

M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=...$ to form an equation in just x and k. It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just y and k.

A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in x collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of a, b and c are used in $b^2-4ac...0$.

FYI The equation in y and k is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe

M1: Attempts to find two critical values for k using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.

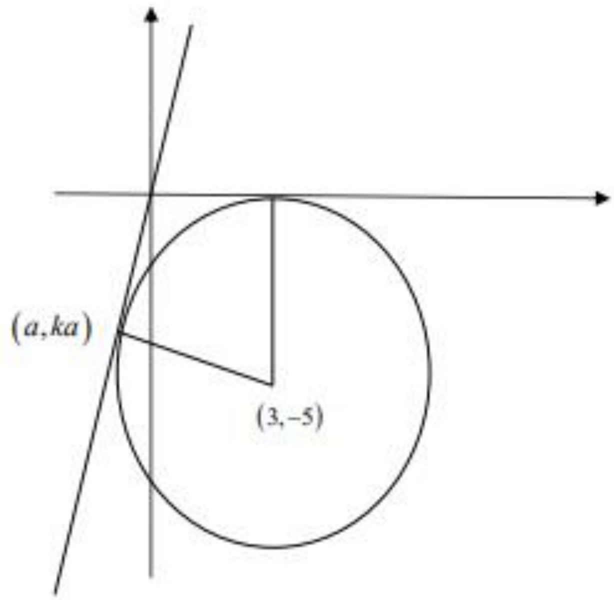
$b^2-4ac=(k-6)^2-4\times(1+k^2)\times9>0\Rightarrow-35k^2-12k>0\Rightarrow k(35k+12)<0$

A1: Deduces $k<0, k>\frac{15}{8}$. This must be in terms of k.

Allow exact equivalents such as $k<0\bigcup k>1.875$

but not allow $0>k>\frac{15}{8}$ or the above with AND, & or $\bigcap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0,0)$, and $(3,-5)$

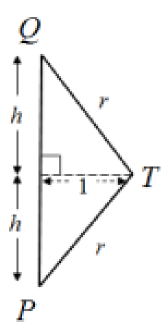


Alt (b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Distance from (a,ka) to $(0,0)$ is $3 \Rightarrow a^2(1+k^2) = 9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k \times \frac{ka+5}{a-3} = -1 \Rightarrow a(1+k^2) = 3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k = \frac{15}{8}$	A1	1.1b
	Deduces $k < 0, k > \frac{15}{8}$	A1	2.2a
		(6)	

May 2017 Mathematics Advanced Paper 1: Pure Mathematics 2

5.

Question number	Scheme	Marks
5 (a)	$x^2 + y^2 - 10x + 6y + 30 = 0$ Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$. Accept $(\pm5, \pm3)$ as indication of this.	M1

	Centre is $(5, -3)$.	A1	**(2)**
(b) Way 1	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give $r = \sqrt{"25" + "9" - 30}$ or $r^2 = "25" + "9" - 30$ (not $30 - 25 - 9$)	M1	
	$r = 2$	A1cao	**(2)**
Or Way 2	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1	
	$r = 2$	A1	**(2)**
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only e.g $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10 \times 4 + 6y + 30 = 0$	M1	
	Solve their quadratic in y and obtain **two** solutions for y	dM1	
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1	**(3)**
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2 - ("5" - 4)^2 = h^2$,	M1	
	Find h and evaluate $"-3" \pm h$. May recognise $(1, \sqrt{3}, 2)$ triangle.	dM1	
	So $y = -3 \pm \sqrt{3}$	A1	**(3)** **[7]**

May 2015 Mathematics Advanced Paper 1: Pure Mathematics 2

6.

Question Number	Scheme	Marks
	Way 1 | **Way 2**	
2 (a)	$(x \mp 2)^2 + (y \pm 1)^2 = k$, $k > 0$ | $x^2 + y^2 \mp 4x \pm 2y + c = 0$	M1
	Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$ | $4^2 + (-5)^2 - 4\times 4 + 2\times -5 + c = 0$	M1
	Obtains $(x-2)^2 + (y+1)^2 = 20$ | $x^2 + y^2 - 4x + 2y - 15 = 0$	A1 **(3)**
	N.B. Special case: $(x-2)^2 - (y+1)^2 = 20$ is not a circle equation but earns M0M1A0	
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)	B1
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$	M1
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$	M1
	So equation is $x - 2y - 14 = 0$ (or $2y - x + 14 = 0$ or other integer multiples of this answer)	A1 **(4)**
b)Way 2	Quotes $xx' + yy' - 2(x+x') + (y+y') - 15 = 0$ and substitutes (4, -5)_	B1
	$4x - 5y - 2(x+4) + (y-5) - 15 = 0$ so $2x - 4y - 28 = 0$ (or alternatives as in Way 1)	M1,M1A1 **(4)**
b)Way 3	Use differentiation to find expression for gradient of circle	
	Either $2(x-2) + 2(y+1)\dfrac{dy}{dx} = 0$ **or** states $y = -1 - \sqrt{20-(x-2)^2}$ so $\dfrac{dy}{dx} = \dfrac{(x-2)}{\sqrt{20-(x-2)^2}}$	B1
	Substitute $x = 4$, $y = -5$ after valid differentiation to give gradient =	M1
	Then as Way 1 above $(y+5) = '\tfrac{1}{2}'(x-4)$ so $x - 2y - 14 = 0$	M1 A1 **(4)**
		[7]

Notes

(a) **M1:** Uses centre to write down equation of circle in one of these forms. There may be sign slips as shown.

M1: Attempts distance between two points **to establish** r^2 (independent of first M1)- allow one sign slip only using distance formula with -5 or -1, usually $(-5-1)$ in 2nd bracket. Must not identify this distance as diameter.

This mark may alternatively (e.g. way 2)be given for substituting (4, -5) into a **correct circle** equation with one unknown

Can be awarded for $r = \sqrt{20}$ or for $r^2 = 20$ stated or implied but not for $r^2 = \sqrt{20}$ or $r = 20$ or $r = \sqrt{5}$

A1: Either of the answers printed or correct equivalent e.g. $(x-2)^2 + (y+1)^2 = (2\sqrt{5})^2$ is A1 but $2\sqrt{5}^2$ (no bracket) is A0 unless there is recovery

Also $(x-2)^2 + (y-(-1))^2 = (2\sqrt{5})^2$ may be awarded M1M1A1as a correct equivalent.

N.B. $(x-2)^2 + (y+1)^2 = 40$ commonly arises from one sign error evaluating r and earns M1M1A0

(b) **Way 1:**

B1: Must be correct answer -2 if evaluated (otherwise may be implied by the following work)

M1: Uses negative reciprocal of their gradient

M1: Uses $y - y_1 = m(x - x_1)$ with (4,-5) and their **changed** gradient **or** uses $y = mx + c$ and (4, -5) with their changed gradient (not gradient of radius) to find c

A1: answers in scheme or multiples of these answers (must have " = 0"). NB Allow $1x - 2y - 14 = 0$

N.B. $(y+5) = '\tfrac{1}{2}'(x-4)$ following gradient of is ½ after errors leads to $x - 2y - 14 = 0$ but is worth B0M0M0A0

Way 2: Alternative method (b) is rare.

Way 3: Some may use implicit differentiation to differentiate- others may attempt to make y the subject and use chain rule

B1: the differentiation must be accurate and the algebra accurate too. Need to take (-) root not (+)root in the alternative

M1: Substitutes into their gradient function but must follow valid accurate differentiation

M1: Must use "their" tangent gradient and $y + 5 = m(x-4)$ but allow over simplified attempts at differentiation for this mark.

A1: As in Way 1

May 2014 Mathematics Advanced Paper 1: Pure Mathematics 2

7.

Question Number	Scheme		Marks
	Mark (a) and (b) together		
9. (a)	$OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{= 14\}$	Uses the addition form of Pythagoras on $6\sqrt{5}$ and 4. Condone missing brackets on $\left(6\sqrt{5}\right)^2$ **(Working or 14 may be seen on the diagram)**	M1
	$y_Q = \sqrt{14^2 - 11^2}$	$y_Q = \sqrt{(\text{their } OQ)^2 - 11^2}$ **Must include √ and is dependent on the first M1 and requires OQ > 11**	dM1
	$= \sqrt{75}$ or $5\sqrt{3}$	$\sqrt{75}$ or $5\sqrt{3}$	A1cso
			[3]
(b)	$(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1: $(x \pm 11)^2 + (y \pm \text{their } k)^2 = 4^2$ **Equation must be of this form and must use *x* and *y* not other letters. *k* could be their last answer to part (a). Allow their $k \neq 0$ or just the letter *k*.**	M1A1
		A1: $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ or $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 4^2$ **NB $5\sqrt{3}$ must come from correct work in (a) and allow awrt 8.66**	
	Allow in expanded form for the final A1 e.g. $x^2 - 22x + 121 + y^2 - 10\sqrt{3}y + 75 = 16$		
			[2]
			Total 5
	Watch out for:		
	(a) $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2} = \sqrt{46}$ M1 $y_Q = \sqrt{46 - 11^2}$ M0 (OQ < 11) $y_Q = \sqrt{75}$ A0 (b) $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ M1A0		

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

8.

Question Number	Scheme	Marks
10. (a)	Equation of form $(x\pm5)^2+(y\pm9)^2=k$, $k>0$	M1
	Equation of form $(x-a)^2+(y-b)^2=5^2$, with values for a and b	M1
	$(x+5)^2+(y-9)^2=25\ =5^2$	A1
		(3)
	$P(8,-7)$. Let centre of circle $=X(-5,9)$	
(b)	$PX^2=(8-\text{"}-5\text{"})^2+(-7-\text{"}9\text{"})^2$ or $PX=\sqrt{(8--5)^2+(-7-9)^2}$	M1
	($PX=\sqrt{425}$ or $5\sqrt{17}$) $PT^2=(PX)^2-5^2$ with numerical PX	dM1
	$PT\ \left\{=\sqrt{400}\right\}=20$ (allow 20.0)	A1 **cso**
		(3) **[6]**
Alternative 2 for (a)	Equation of the form $x^2+y^2\pm10x\pm18y+c=0$	M1
	Uses $a^2+b^2-5^2=c$ with their a and b or substitutes (0, 9) giving $+9^2\pm2b\times9+c=0$	M1
	$x^2+y^2+10x-18y+81=0$	A1
		(3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17},11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first)	M1
	M1: Use either of the correct points with P (8, -7) and distance between two points formula	dM1
	A1: 20	A1cso
		(3)
Alternative 3 for (b)	Substitutes (8, -7) into circle **equation** so $PT^2=\ 8^2+(-7)^2+10\times8-18\times(-7)+81$	M1
	Square roots to give $PT\ \left\{=\sqrt{400}\right\}=20$	dM1A1
		(3)

	Notes for Question 10
(a)	**The three marks in (a) each require a circle equation – (see special cases which are not circles)** M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k>0$ or a positive value) M1: Uses $r=5$ to obtain RHS of circle equation as 25 or 5^2 A1: correct circle equation in any equivalent form **Special cases** $(x\pm5)^2+(x\pm9)^2=(5^2)$ is **not a circle** equation so M0M0A0 Also $(x\pm5)^2+(y-9)=(5^2)$ **And** $(x\pm5)^2-(y\pm9)^2=(5^2)$ **are not circles** and gain M0M0A0 **But** $(x-0)^2+(y-9)^2=5^2$ gains M0M1A0
(b)	M1: Attempts to find distance from their **centre of circle** to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X, as centre was not labelled in the question. N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0. dM1**:** Applies the **subtraction** form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from **centre to *P***) or uses appropriate complete method involving trigonometry A1: 20 cso

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

9.

Question Number	Scheme		Marks
5.			
(a)	**Parts (i) and (ii) are likely to be solved together so mark as one part**		
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2=-195+100+144 \Rightarrow r=...$		M1
	Completes the square for both x and y in an attempt to find r. $(x\pm"10")^2 \pm a$ and $(y\pm"12")^2 \pm b$ and $+195=0,(a,b\neq 0)$ Allow slips in obtaining their r^2 but must find square root		
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r=7$	Not $r=\pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r=\sqrt{(\pm"10")^2+(\pm"12")^2-c}$		M1
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r	A1
	$r=7$		A1
			(5)
	Note that although the marks for the centre are B marks, they do need to come from correct work. E.g. $(x+10)^2$, $(y+12)^2$ giving a centre of (10, 12) scores B0 B0 but could score the M1A1ftA1ft for the radius as a special case. Similarly $(x+10)^2$, $(y-12)^2$ giving a centre of (-10, 12) scores B0 B1, $(x-10)^2$, $(y+12)^2$ giving a centre of (10, -12) scores B1 B0 but both could score M1A1ftA1ft for the radius as a special case also.		
(b)	$MN=\sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right)=25$		A1
			(2)
(c)	$NP=\sqrt{("25"^2-"7"^2)}$	$NP=\sqrt{(MN^2-r^2)}$	M1
	$NP=\sqrt{(25^2+7^2)}$ is M0 (Quite common)		
	$NP\left(=\sqrt{576}\right)=24$		A1
			(2)
(c) Way 2	$\cos(NMP)=\dfrac{7}{"25"} \Rightarrow NP="25"\sin(NMP)$	Correct strategy for finding NP	M1
	$NP=24$		A1
			(2)
			[9]

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10.

Question number	Scheme	Marks
3 (a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	Obtain $(x-10)^2$ **and** $(y-8)^2$	A1
	Centre is (10, 8). N.B. This may be indicated on **diagram only** as (10, 8)	A1 (3)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25 (= r^2)$ or $(r^2 =)"100"+"64"-139$	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	**A1** **(2)**
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$	**M1**
	e.g $x=13 \Rightarrow (13-10)^2+(y-8)^2=25 \Rightarrow (y-8)^2=16$ so $y=$ or $13^2+y^2-20\times13-16y+139=0 \Rightarrow y^2-16y+48=0$ so $y=$	
	$y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	**A1, A1** (3)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)	**M1**
	Perimeter $PTQ = 2r +$ their **arc** PQ (Finding perimeter of triangle is M0 here)	**M1**
	$= 19.275$ or 19.28 or 19.3	**A1** (3)
		11 marks
Alternatives (a)	*Method 2:* From $x^2+y^2+2gx+2fy+c=0$ centre is $(\pm g, \pm f)$ Centre is $(-g, -f)$, **and so centre is** (10, 8).	M1 A1, A1
OR	*Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	*Method 2*: Using $\sqrt{g^2+f^2-c}$ or $(r^2 =)"100"+"64"-139$ $r = 5$ *	M1 A1
OR	*Method 3:* Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2+(12-8)^2}$ $r = 5$ *	**M1** **A1 cao** (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2-(13-"10")^2=h^2$, then evaluate $"8 \pm h"$ - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	**M1**

Notes (a) (b)	**Mark (a) and (b) together** **M1** as in scheme and can be <u>implied</u> by $(\pm 10, \pm 8)$ **. Correct centre (10, 8) implies M1A1A1** **M1** for a correct method leading to $r = \ldots$, or $r^2 =$ "100"+"64"−139 (not 139 – "100" –" 64") or for using equation of circle in $(x \pm 10)^2 + (y \pm 8)^2 = k^2$ form to identify $r=$ **3rd A1** $r = 5$ **(NB This is a given answer so should follow** $k^2 = 25$ or $r^2 = 100 + 64 - 139$ **)** **Special case**: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow **M1A1** for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector

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11.

Question number	Scheme	Marks
2	The equation of the circle is $(x+1)^2 + (y-7)^2 = (r^2)$ The radius of the circle is $\sqrt{(-1)^2 + 7^2} = \sqrt{50}$ or $5\sqrt{2}$ or $r^2 = 50$ So $(x+1)^2 + (y-7)^2 = 50$ or equivalent	M1 A1 M1 A1 (4) **4**
Notes	**M1** is for this expression on left hand side– allow *errors in sign* of 1 and 7. **A1** correct signs (just LHS) **M1** is for Pythagoras or substitution into equation of circle to give r or r^2 Giving this value as diameter is **M0** **A1**, cao for cartesian equation with numerical values but allow $(\sqrt{50})^2$ or $(5\sqrt{2})^2$ or any exact equivalent A correct answer implies a correct method – so answer given with no working earns all four marks for this question.	
Alternative method	Equation of circle is $x^2 + y^2 \pm 2x \pm 14y + c = 0$ Equation of circle is $x^2 + y^2 + 2x - 14y + c = 0$ Uses (0,0) to give $c = 0$, **or** finds $r = \sqrt{(-1)^2 + 7^2} = \sqrt{50}$ or $5\sqrt{2}$ or $r^2 = 50$ So $x^2 + y^2 + 2x - 14y = 0$ or equivalent	M1 A1 M1 A1

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12.

Question Number	Scheme		Marks
4.	$x^2 + y^2 + 4x - 2y - 11 = 0$		
(a)	$\{\underline{(x+2)^2 - 4} + \underline{\underline{(y-1)^2 - 1}} - 11 = 0\}$	$(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$.	$(-2, 1)$.	A1 **cao** **[2]**
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$	$r = \sqrt{11 \pm "1" \pm "4"}$	M1
	So $r = \sqrt{11+1+4} \Rightarrow r = 4$	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 **[2]**
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$	Putting $x = 0$ in C or their C.	M1
		$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \dfrac{2 \pm \sqrt{48}}{2}\right\}$	Attempt to use formula or a method of completing the square in order to find $y = \ldots$	M1
	So, $y = 1 \pm 2\sqrt{3}$	$1 \pm 2\sqrt{3}$	A1 **cao cso** **[4]**
			8

	Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks. **Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.**
(a)	M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $\underline{(x \pm 2)^2 \pm \alpha}$, $\alpha \neq 0$ or $\underline{\underline{(y \pm 1)^2 \pm \beta}}$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.
(b)	M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 \pm "1" \pm "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14. <u>**Note:**</u> $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0. <u>**Alternative:**</u> M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this method mark. $(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8$ scores M0A0, but $r = \sqrt{16} = 8$ scores M1A1 isw.
(c)	1st M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y **in any form** which can be implied by later working. 2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise. 2nd A1: Need exact pair in simplified surd form of $\{y =\}\ 1 \pm 2\sqrt{3}$. This mark is also cso. Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2nd A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3}, 0)$. Any incorrect working in (c) gets penalised the final accuracy mark. So, <u>**beware:**</u> incorrect $(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0. <u>**Special Case for setting y = 0:**</u> Award SC: M0A0M1A0 for an attempt at applying the formula $x = \dfrac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \dfrac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15}\right\}$ Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. <u>**Special Case:**</u> For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.

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13.

Question Number	Scheme		Marks
9.			
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2+(1-6)^2$ or $\sqrt{(8-3)^2+(1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2+(11-6)^2$ or $\sqrt{(-2-3)^2+(11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2+(y-6)^2=50$ $\left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x\pm 3)^2+(y\pm 6)^2=k$, k is a positive value.	M1
		$(x-3)^2+(y-6)^2=50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$,} $\underline{(10-3)^2+(7-6)^2=50}$, {so the point lies on C.}		B1
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y-7=-7(x-10)$	$y-7=(\text{their gradient})(x-10)$	M1
	$y=-7x+77$	$y=-7x+77$ or $y=77-7x$	A1 cao
			(4)
			[10]

	Notes
(a)	Alternative method: $C\left(-2+\frac{8--2}{2}, 11+\frac{1-11}{2}\right)$ or $C\left(8+\frac{-2-8}{2}, 1+\frac{11-1}{2}\right)$
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\frac{(-2-8)^2+(11-1)^2}{2}$ Award 1st M1A1 for $\frac{(-2-8)^2+(11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2+(11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.
(c)	B1 awarded for correct verification of $\underline{(10-3)^2+(7-6)^2=50}$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x=10$ in C to find $y=7$ or substitute $y=7$ in C to find $x=10$.

Question Number	Scheme	Marks
(d)	2^{nd} M1 mark also for the complete method of applying $7 =$ (their gradient)(10) $+ c$, finding c. **Note**: Award 2^{nd} M0 in (d) if their numerical gradient is either 0 or ∞. **<u>Alternative:</u>** For first two marks (differentiation): $2(x-3)+2(y-6)\frac{dy}{dx}=0$ (or equivalent) scores B1. 1^{st} M1 for substituting **both** $x=10$ and $y=7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted "differentiation" is not "implicit".) **<u>Alternative</u>:** $(10-3)(x-3)+(7-6)(y-6)=50$ scores B1M1M1 which leads to $y=-7x+77$.	

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14.

Question Number	Scheme	Marks
10	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$	M1 A1
	$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive <u>value</u>)	M1
	$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100)	A1
	(Answer only scores full marks)	(4)
	(b) (Gradient of radius =) $\frac{7-1}{10-2}=\frac{6}{8}$ (or equiv.) Must be seen in part (b)	B1
	Gradient of tangent $=\frac{-4}{3}$ (Using perpendicular gradient method)	M1
	$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)	M1
	$y-7=\frac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks)	A1ft
	$\{3y=-4x+61\}$ (N.B. The A1 is only available as <u>ft</u> after B0) The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here. The equation must at some stage be <u>exact</u>, not, e.g. $y=-1.3x+20.3$	
		(4)
	(c) $\sqrt{r^2-\left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.	M1
	$=\sqrt{10^2-5^2}$ or numerically exact equivalent	A1
	$PQ\left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	A1
		(3) **11**

	(b) 2^{nd} M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). Alternative: 2^{nd} M: Using (10, 7) and an m value in $y = mx + c$ to find a value of c. (b) Alternative for first 2 marks (differentiation): $2(x-2)+2(y-1)\frac{dy}{dx}=0$ or equiv. B1 Substitute $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$ M1 (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). (c) Alternatives: To score M1, must be a fully correct method to obtain $\frac{1}{2}PQ$ or PQ. 1^{st} A1: For alternative methods that find PQ directly, this mark is for an exact numerically correct version of PQ.	

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15.

Question Number	Scheme	Marks
Q8 (a)	$N(2, -1)$	B1, B1 (2)
(b)	$r=\sqrt{\frac{169}{4}}=\frac{13}{2}=6.5$	B1 (1)
(c)	Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\frac{x_1+x_2}{2}=2$ then solve	M1
	To obtain $x_1 = -4$, $x_2 = 8$	A1ft A1ft
	Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A *then* $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$	M1
	So $y_2 = y_1 = -3.5$	A1 (5)
(d)	Let $A\hat{N}B = 2\theta \Rightarrow \sin\theta = \frac{6}{"6.5"} \Rightarrow \theta = (67.38)...$	M1
	So angle ANB is 134.8 *	A1 (2)
(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta = \frac{AP}{"6.5"}$	M1
	Therefore $AP = 15.6$	A1cao (2)
		[12]

(a) (b)	B1 for 2 (α), B1 for –1 B1 for 6.5 o.e.	
(c)	1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for $\alpha - 6$ and $\alpha + 6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for –3.5 only.	
(d)	M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong *y*.** (cos ANB = $\frac{"6.5"^2 + "6.5"^2 - 12^2}{2 \times "6.5" \times "6.5"}$ =-0.704) A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find AP <u>Alternative Methods</u> N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle BAP = 67.38 or some variation. Eg $\frac{AP}{\sin 67.4} = \frac{12}{\sin 45.2}$, $AP = \frac{6}{\sin 22.6}$ or $AP = \frac{6}{\cos 67.4}$ are each worth M1	